1/

What to talk about in my class on April 18, 2024 (Thu):

- relative G-modular category. ← (Turaev "Homotopy quantum field theory (2010)")

  an invariant of a T-admissible triple (M, T, ω)

  Costantino-Geer-Patureau invariant (2012)

  extended to TQFT by

  Blanchet-C-G-P (2014)

  ↑ cobordism categories of surfaces and 3-manifolds equipped with flat G-connection.

  use "universal construction"

  $V : Cob \to GrVect$

  get mapping class group representation

  where Dehn twist is of infinite order

  (which is always of finite order in WRT)

"Admissible skein modules"

[Costantino-Geer-Patureau (2023)]

$\mathcal{C}$ ribbon $\Bbbk$-category with ideal $\mathcal{I}$

<u>$\mathcal{I}$-admissible ribbon graph</u> in Y

is a $\mathcal{C}$-colored ribbon graph in Y where
each connected component of Y contains at least one edge colored with an object of $\mathcal{I}$.

A formal linear combination $\sum_{1\le i\le m} a_i \Gamma_i$ is an <u>$\mathcal{I}$-skein relation</u>

(Γ_i ~ $\mathcal{I}$-admissible ribbon graphs in Y)

if there exists a box B in Y and Γ_i' isotopic to Γ_i $(i=1,\cdots,m)$

transverse to the box B and identical outside the box,

such that

(1) $\sum_i a_i F(\Gamma_i' \cap B) = 0$ as a morphism in $\mathcal{C}$,

(2) each Γ_i' has an $\mathcal{I}$-colored edge not completely contained in B

<u>Def</u> The <u>admissible skein module</u> $S_{\mathcal{I}}(Y)$

is the $\Bbbk$-span of $\mathcal{I}$-admissible ribbon graphs in Y
modulo (the span of) $\mathcal{I}$-skein relations.

↑ actually generated (spanned) by ribbon graphs each of whose edges are colored by an obj in $\mathcal{I}$.

3/

$S_I(\Sigma)$ is a (non-unital if $I \neq \mathcal{C}$) associative algebra

Thm ([CGP(2023)])

Let $\mathcal{C}$ be an (essentially small) pivotal $\Bbbk$-category

and let I be an ideal of $\mathcal{C}$.

Then,

$$S_I(D^2)^* \cong \{\text{right m-traces on } I\} \cong \{\text{left m-traces on } I\},$$

and

$$S_I(S^2)^* \cong \{\text{m-traces on } I\}$$

Cor Let $\mathcal{C}$ be an (essentially small) ribbon $\Bbbk$-cat. and I an ideal of $\mathcal{C}$.

Then, $S_I(B^3)^* \cong S_I(S^3)^* \cong \{\text{m-traces on } I\}$

[C-G-Haioun-P (2023)] constructs a non-compact (3+1)d TQFT

where the state spaces are admissible skein modules.

(under some assumptions on $\mathcal{C}$)

including existence of non-degenerate m-trace

and a chromatic morphism for a projective generator